Simplified Quartessence Cosmology

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We propose a new class of accelerating world models unifying the cosmological dark sector (dark matter and dark energy). All the models are described by a simplified version of the Chaplygin gas Quartessence cosmology. It is found that even for $\Omega_k \neq 0$, this Quartessence scenario depends only on a pair of parameters which can severely be constrained by the cosmological tests. As an example we perform a joint analysis involving the latest SNe type Ia data and the recent Sloan Digital Sky Survey measurement of baryon acoustic oscillations. In our analysis we have considered separately the SNe type Ia gold sample measured by Riess *et al.* (2004) and the Supernova Legacy Survey (SNLS) from Astier *et al.* (2006). At 95.4% (c.l.), we find for BAO + *gold* sample, $\alpha = 0.82^{+0.04}_{-0.06}$ and $\Omega_{Q4} = 1.08^{+0.25}_{-0.31}$ while BAO + SNLS analysis provides $\alpha = 0.83^{+0.03}_{-0.05}$ and $\Omega_{Q4} = 1.11^{+0.21}_{-0.26}$. The best fit for this simplified Quartessence scenario is a spatially closed Universe and with the same number of parameters, the $\chi^2 = 174.6$ is slightly smaller than the one of the flat concordance model (ΛCDM).

Keywords: Cosmology, Dark Energy, Classical Tests

I. INTRODUCTION

The most plausible picture for the observed Universe seems to be represented by a nearly flat scenario dominated by cold dark matter (CDM) and a relativistic component endowed with large negative pressure, usually named dark energy [1, 2, 3]. Although having different status from a theoretical and observational viewpoints, the actual nature of these dominant components remains unknown until the present. Therefore, in certain sense, one may say that the modern general relativistic cosmology is plagued with the so-called cosmological "dark sector problem".

Recently, many cosmological models driven by dark matter and dark energy have been proposed in the literature aiming at explaining the late time cosmic acceleration and other recent observational results [4, 5]. Among these scenarios, a very interesting one was suggested by Kamenshchik *et al.* [6] and developed by Bilić *et al.* [7] and Bento *et al.* [8]. It corresponds to a class of world models dominated by an exotic fluid, named Chaplygin gas (C-gas), which can be macroscopically characterized by the equation of state (EoS)

$$p_C = -A/\rho_C^{\alpha}, \tag{1}$$

where $\alpha = 1$ and A is a positive constant related to the present-day Chaplygin adiabatic sound speed, $v_s^2 = \alpha A/\rho_{C_o}^{1+\alpha}$ (ρ_{C_o} stands for the current C-gas density).

The above equation for $\alpha \neq 1$ constitutes a generalization of the original C-gas EoS proposed by Bento *et al.* in Ref. [8]. One of its fundamental features comes from the fact that the C-gas becomes pressureless at high redshifts, which suggests a possible unification scheme for the cosmological "dark sector" (CDM plus dark energy). Scenarios driven by a C-gas (without an extra CDM component) are usually termed quartessence models and have been largely explored in the literature [9].

In most of these quartessence analyses, besides the present value of the C-gas density parameter (Ω_C), the above barotropic EoS implies that one needs to constrain two additional free parameters, namely, A and α since the baryonic density (Ω_b) may be fixed a priori by using, for instance, nucleosynthesis or the recent Cosmic Microwave Background (CMB) observations [10]. Therefore, in the context of a general Friedman-Robertson-Walker (FRW) cosmologies quartessence scenarios require at least 3 parameters to be constrained by the data (see for instance, Bertolami *et al.* [12]). In other words, there are so many parameters to be constrained by the data, that a high degree of degeneracy on the parametric space becomes inevitable. The common solution in the literature to reduce the number of free parameters (motivated by current CMB results) is to assume a flat geometry, i.e., $\Omega_{Q4} = 1 - \Omega_b$, where Ω_{Q4} and Ω_b stand, respectively, for the baryons and C-gas density parameters.

More recently, some generalizations of the original C-gas [13, 14, 15, 16, 17], or even of its extended version [18] have appeared in literature. In these cases, the number of free parameters is usually increased, and, as consequence, the models become mathematically richer although much less predictive from a physical viewpoint. In a recent paper (from now on paper I) we took the opposite way, that is, we have proposed a large set of cosmologies driven by dark energy plus a CDM component where the dark energy component was represented by a simplified version of the Chaplygin gas [19].

In this work we explore the results of paper I now for a Quartessence version. We discuss what we believe to be the simplest Quartessence scenario, that is, the one with

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the smallest number of free parameters. As we shall see, by an additional physical condition, the allowed range of the α free parameter is also restricted a priori, which makes not only the relevant parametric space bidimensional - even for nonflat spatial sections - but also (and more important) the model can be more easily discarded or confirmed by the present set of observations since the range of its free parameter is physically limited from causality considerations. We test the viability of this simplified Quartessence approach by discussing the constraints imposed from current SNe Ia observations (using both the gold sample and LSNL data set) and Large Scale Structure (LSS) data.

II. A SIMPLIFIED QUARTESSENCE SCENARIO

Let us now consider that the geometrical properties of the observed Universe are described by the general FRW line element

$$ds^2 = dt^2 - a(t)^2 \left(\frac{dr^2}{1-kr^2} + r^2 d\Sigma^2 \right), \quad (2)$$

where $a(t)$ is the scale factor, $d\Sigma^2$ is the area element on the unit 2-sphere, $k = 0, \pm 1$ is the curvature parameter and we have adopted the metric signature convention (+,−,−,−). Throughout this paper we adopt units such that $c = 1$. The matter content of the Universe is assumed to be composed of a baryonic component plus the quartessence C-gas fluid.

Since each component is separately conserved, one may integrate out the energy conservation for the C-gas, $\dot{\rho}_C = -3H(\rho_C + p_C)$, to obtain the following expression for its energy density [8, 9, 20]

$$\rho_C = \rho_{C_o} \left[A_s + (1 - A_s) a^{3(1+\alpha)} \right]^{\frac{1}{1+\alpha}}, \quad (3)$$

where $A_s = A/\rho_{C_o}^{1+\alpha}$ is a convenient dimensionless constant (as usual, the subscript "0" denotes present-day quantities). In the background defined by (2), the Friedmann equation for a conserved C-gas plus the baryonic component reads

$$\mathcal{H} = \left[\Omega_{\rm b} (\frac{a_0}{a})^3 + \Omega_{\rm Q4} f(A_s, \alpha) + \Omega_k (\frac{a_0}{a})^2 \right]^{1/2}, \quad (4)$$

where $\mathcal{H} \equiv H/H_0$ (H is the Hubble parameter), the function $f(A_s, \alpha)$ is given by $f(A_s, \alpha) = [A_s + (1 - A_s)(\frac{a_0}{a})^{3(\alpha+1)}]^{\frac{1}{\alpha+1}}$ and Ω_k is the fractional contribution of the spatial curvature to $\mathcal{H}$. Note that, besides the Hubble parameter H_0, we still have 3 additional parameters in this case ($\alpha, A_s, \Omega_{\rm Q4}$), since the baryonic contribution is defined to be $\simeq 4.4\%$ from current CMB experiments [11]. This is the standard treatment. Therefore, the important aspect to be discussed at this point is how to reduce the quartessence C-gas parameters based on reasonable physical requirements?

In order to answer the above question, we follow the arguments of Ref. [21]. Note that the C-gas adiabatic sound speed reads

$$v_s^2 = \frac{dp}{d\rho} = \alpha A/\rho_C^{1+\alpha}, \quad (5)$$

which must be positive definite for a well-behaved gas (zero in the limit case of dust). Note also that the present-day C-gas adiabatic sound speed is $v_{so}^2 = \alpha A/\rho_{C_o^{1+\alpha}}$, or still

$$v_{so}^2 = \alpha A/\rho_{C_o}^{1+\alpha} = \alpha A_s. \quad (6)$$

Therefore, from the above equation one clearly see that if the parameter A_s is a function of the index α, i.e., $A_s \to A_s(\alpha)$, the number of free parameters is naturally reduced, and, as an extra bonus, the positiviness of v_s^2 at any time, as well as its thermodynamic stability, is naturally guaranteed. Clearly, among many possibilities the simplest choice is $A_s \propto \alpha$, which we assume in this paper. In this case, $v_{so}^2 = \alpha^2$, or more generally, $v_s^2 = \alpha^2(\rho_{Co}/\rho)^\alpha$. Note also that, since the light speed is a natural cutoff for the sound propagation, it follows that $v_{so} = |\alpha| \leq 1$, thereby restricting α to the interval [-1,1]. An additional constraint can still be imposed to this parameter. In fact, with $A_s \propto \alpha$, the simplified C-gas EoS (1) becomes

$$p_C = -\alpha \rho_{Co} \left(\frac{\rho_{Co}}{\rho_C} \right)^\alpha, \quad (7)$$

so that a negative pressure is obtained only for positive values of α. In other words, this accounts to saying that the combined requirements from causality along with the observed accelerating stage of the Universe naturally restrict the parameter α to the interval $0 < \alpha \leq 1$ [21].

Note that the simplified quartessence component preserves the unifying character of the original C-gas, i.e., it behaves as a pressureless fluid (non-relativistic matter) at high-z while, at late times, it approaches the quintessence behavior, which now is fully characterized by the α parameter. However, note also that, even in this limiting case, the sound speed is positive.

In this simplified approach, Eq.(4) is rewritten as

$$\mathcal{H} = \left[\Omega_{\rm b} (\frac{a_0}{a})^3 + \Omega_{\rm Q4} g(\alpha) + \Omega_k (\frac{a_0}{a})^2 \right]^{1/2}, \quad (8)$$

where the function $g(\alpha)$ is simply given by $g(\alpha) = [\alpha + (1 - \alpha)(\frac{a_0}{a})^{3(\alpha+1)}]^{\frac{1}{\alpha+1}}$, so that the only remaining parameters to be determined in this unified dark matter/energy scenario are α and $\Omega_{\rm Q4}$. In what follows, we confront this simplified quartessence scenario with some the most recent SNe Ia and Large Scale Structure (LSS) data.

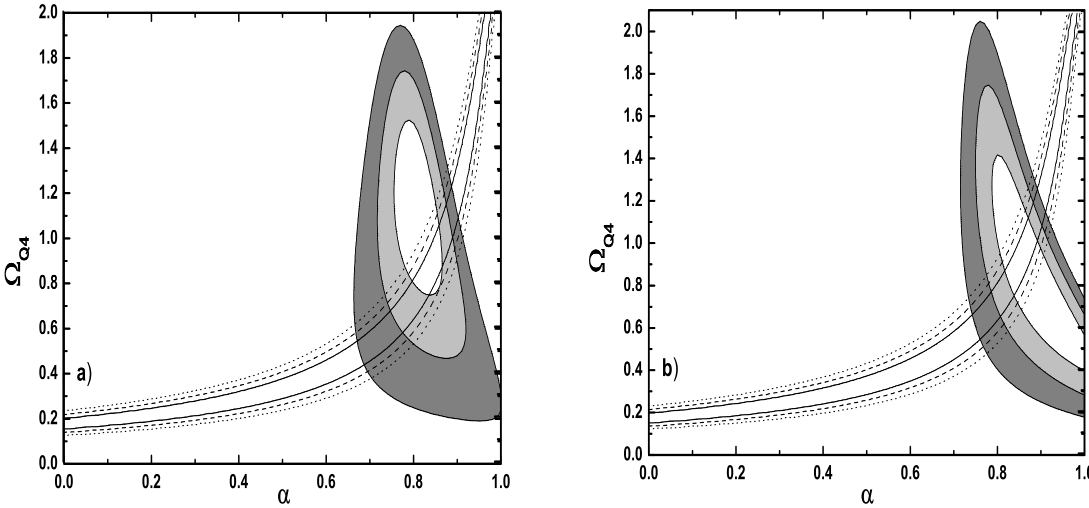

FIG. 1: In panels (a) and (b) we display the predictions based on the simplified quartessence scenario characterized by the pair (Ω_{Q4}, α). The contours in panels (a) and (b) were obtained using the SNe data from HZS (gold) and LNLS samples, respectively. The dotted and solid lines represent the contours from SDSS BAO measurements alone. Note that they are almost orthogonal to the parameter space defined by the supernova data.

III. OBSERVATIONAL CONSTRAINTS

A. SNe Ia

Let us first investigate the bounds arising from SNe Ia observations on the SC-gas scenario described above. To this end we use the most recent SNe Ia observations, namely, the High-Z SN Search (HZS) Team [2] and the Supernova Legacy Survey (SNLS) Collaboration data [3] (we refer the reader to the original references and to Paper I for more details on these data sets).

The predicted distance modulus for a supernova at redshift z, given a set of parameters $\mathbf{p}$, is

$$\mu_p(z|\mathbf{p}) = m - M = 5\log d_L + 25, \tag{9}$$

where m and M are, respectively, the apparent and absolute magnitudes, the complete set of parameters is $\mathbf{p} \equiv (H_o, \Omega_{Q4}, \alpha)$ and d_L stands for the luminosity distance (in units of megaparsecs),

$$d_L = H_o^{-1}(1+z)\frac{1}{\sqrt{|\Omega_k|}}\xi\left(\sqrt{|\Omega_k|}\int_{x'}^{1}\frac{dx}{x^2\mathcal{H}(x;\mathbf{p})}\right), \tag{10}$$

with $x' = (1+z)^{-1}$, $\mathcal{H}(x;\mathbf{p})$ the expression given by Eq. (8), and the function $\xi(x)$ is defined as $\xi(x) = \sin(x)$ for a closed universe, $\xi(x) = \sinh(x)$ for an open universe and $\xi(x) = x$ for a flat universe.

We estimated the best fit to the set of parameters $\mathbf{p}$ by using a χ^2 statistics

$$\chi^2 = \sum_{i=1}^{N}\frac{\left[\mu_p^i(z|\mathbf{p}) - \mu_o^i(z|\mathbf{p})\right]^2}{\sigma_i^2}, \tag{11}$$

with the parameters Ω_{Q4} and α spanning the interval [0,1] in steps of 0.01. In the above expression, $N = 157$ and 115 for *gold* and SNLS samples, respectively, $\mu_p^i(z|\mathbf{p})$ is given by Eq. (9), $\mu_o^i(z|\mathbf{p})$ is the extinction corrected distance modulus for a given SNe Ia at z_i, and σ_i is the uncertainty in the individual distance moduli. In our analysis, H_0 is considered a *nuisance* parameter so that we marginalize over it.

In Figures (1a) and (1b) we show the results of our statistical analysis. Contours of constant likelihood (99.73%, 95.4% and 68.3%) are shown in the parametric space $\alpha - \Omega_{Q4}$. Panel (1a) displays the results for the HZS *gold* sample. Note that although degenerate in Ω_{Q4}, the parameter α is now considerably more restricted than in the standard C-gas approach (see, e.g., Fig. 4 of Ref. [21]). In particular, note also that for any value of the C-gas density parameter, models with $\alpha \lesssim 0.63$ are ruled out at 99.73% level. The best-fit model for this analysis occurs for $\Omega_{Q4} = 1.15$ and $\alpha = 0.81$ with $\chi^2_{\rm min} = 174.6$ ($\chi^2_{\rm min}/\nu = 1.127$, where $\nu \equiv$ degrees of freedom). At 95.4% c.l. we also find $0.62 \leq \Omega_{Q4} \leq 1.65$ and $0.74 \leq \alpha \leq 0.90$. Panel (1b) shows a similar analysis for the SNLS data. The best-fit parameters in this case are $\Omega_{Q4} = 0.85$ and $\alpha = 0.88$ with $\chi^2_{\rm min} = 113.5$ (with $\chi^2_{\rm min}/\nu \simeq 1.0$). The SNLS sample also imply $0.32 \leq \Omega_{Q4} \leq 1.62$ and $0.77 \leq \alpha \leq 1.0$ at 95.4% (c.l.).

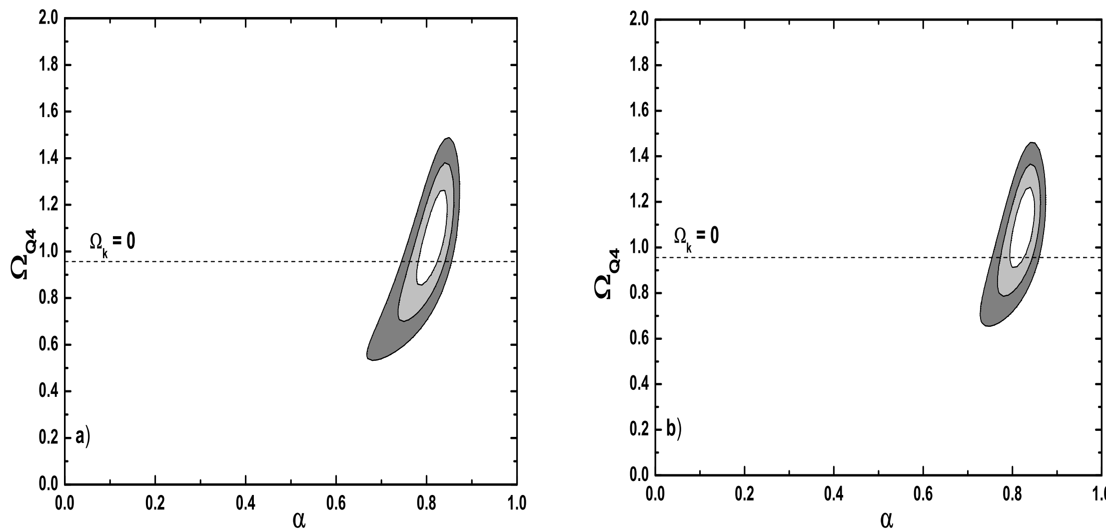

FIG. 2: Contours on the space parameter $(\Omega_{\rm Q4}, \alpha)$ from a joint analysis involving SNe type Ia and the Sloan Sky Digital Survey (SDSS) baryon acoustic oscillations. The corresponding 68.3%, 95.4% and 99.73% c.l. are shown, and, for both panels the horizontal dashed lines represent the flat models. In panel (a) we display the results for the SNe sample observed by Riess *et al.* (2004). In panel (b) we show the contours to the SNe sample observed by Astier *et al.* (2006). For SNe data alone, if α is greater than 0.65, all values of $\Omega_{\rm Q4}$ are ruled out by the two samples. Constraints from BAO contribute to increase the allowed values of $\Omega_{\rm Q4}$.

B. SNe Ia + LSS analysis

In order to break possible degeneracies in the $\Omega_{\rm Q4} - \alpha$ space, we study now the the joint constraints on this plane from SNe Ia and LSS data. For the LSS data, we use the recent measurements of the BAO peak in the large scale correlation function detected by Eisenstein et al. [22] using a large sample of luminous red galaxies from the SDSS Main Sample. The SDSS BAO measurement provides $\mathcal{A} = 0.469(n_S/0.98)^{-0.35} \pm 0.017$, with $\mathcal{A}$ defined as

$$\mathcal{A} \equiv \frac{\Omega_{\rm M}^{1/2}}{\mathcal{H}(z_{\rm BAO};\mathbf{p})^{1/3}}[\frac{1}{z_{\rm BAO}\sqrt{|\Omega_k|}} \xi\left(\sqrt{|\Omega_k|}\Gamma(z_{\rm BAO};\mathbf{p})\right)]^{2/3}, \quad (12)$$

where $z_{\rm BAO} = 0.35$, $\mathcal{H}(z_{\rm BAO};\mathbf{p})$ is given by Eq. (8), and we take the scalar spectral index $n_S = 0.95$, as given by Spergel et al. [11]. In the above expression, $\Gamma(z_{\rm BAO})$ is the dimensionless comoving distance to $z_{\rm BAO}$, and $\Omega_M = \Omega_b + (1-\alpha)\Omega_{Q4}$. The total matter contribution was derived by using the separation proposed in Ref. [23].

As shown in Figures (1a) and (1b) the dotted lines representing the constraints from SDSS BAO measurements on the parameter space $\Omega_{\rm Q4} - \alpha$ are approximately orthogonal to those arising from SNe Ia data, which indicates that possible degeneracies in this plane may be broken by this combination of observational data. Figures (2a) and (2b) show the results of our joint analyses for the BAO+*gold* and BAO+SNLS samples, respectively. Note that the available parametric plane in both cases are considerably reduced relative to the former analyses (Figs. 2a and 2b). For the BAO+*gold* sample we find $\Omega_{\rm Q4} = 1.08^{+0.25}_{-0.31}$ and $\alpha = 0.82^{+0.04}_{-0.06}$ at 95.4% (c.l.). This latter best-fit scenario corresponds to an accelerating universe with $q_0 \simeq -0.67$, a total age of the Universe of $t_o \simeq 10h^{-1}$ Gyr, and a D/A redshift transition (from deceleration to acceleration) $z_{\rm D/A} \simeq 0.46$. At 95.4% c.l., the BAO+SNLS analysis also provides $\alpha = 0.83^{+0.03}_{-0.05}$ and $\Omega_{\rm Q4} = 1.11^{+0.21}_{-0.26}$.

IV. FINAL REMARKS

A considerable amount of observational evidence suggests that the current evolution of our Universe is fully dominated by two dark components, the so-called dark matter and dark energy. The nature of these components, however, is a tantalizing mystery at present, and it is not even known if they constitute two separate substances. In this paper, we have argued that one of the candidates for a unifying dark matter/dark energy scenario, a C-gas quartessence whose EoS is given by Eq. (1), may have a very simplified description. We have postulated that if A_s is a function of the index α the resulting FRW cosmology (with arbitrary curvature) can be completely described only by a pair of parameters (α, $\Omega_{\rm Q4}$). For the sake of simplicity, we have considered $A_s \propto \alpha^n$ with

$n = 1$.

By considering this class of parameterization we have investigated the constraints from current SNe Ia and LSS data. We have shown that, differently from the original C-gas models (in which the value of the index α is completely degenerated) a joint analysis involving these data sets restricts considerably the $\Omega_{\rm Q4} - \alpha$ parametric space [Figs. (2a) and (2b)] with $\alpha = 0.83^{+0.03}_{-0.05}$ and $\Omega_{\rm Q4} = 1.11^{+0.21}_{-0.26}$. At the level of SNe Ia data and BAO, we may conclude that this class of quartessence scenario passes this combination of tests, thereby providing an interesting possibility to a dark matter/dark energy unification. It is worth noticing that the best-fit for this simplified quartessence scenario corresponds to a spatially closed universe and, with the same number of parameters, the $\chi^2_{min} = 174.6$ is slightly smaller than the one of the flat concordance model (ΛCDM). Naturally, it should be interesting to check if current CMB and other independent observations can confirm or discard the simplified quartessence scenario proposed here.

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